Exercise 7.1

1. Find the distance between the following pairs of points:

(i) $(2,3),(4,1)$

Sol. The given points are $A(2,3)$ and $B(4,1)$

$A(2,3)$ $B(4,1)$

$\therefore$ Distance $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-2)^2+(1-3)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$ Ans.

(ii) $(-5,7),(-1,3)$

Sol. The given points are $P(-5,7)$ and $Q(-1,3)$

$P(-5,7)$ $Q(-1,3)$

$\therefore$ Distance $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[-1-(-5)]^2+(3-7)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$ Ans.

(iii) $(a,b),\ (-a,-b)$

Sol. The given points are $C(a,b)$ and $D(-a,-b)$

$C(a,b)$ $D(-a,-b)$

$\therefore$ Distance $CD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[-a-(-a)]^2+[-b-(-b)]^2}$

$= \sqrt{4a^2+4b^2}$

$= 2\sqrt{a^2+b^2}$ Ans.

2. Find the distance between the points $A(0,0)$ and $B(36,15)$. Can you now find the distance between the two towns A and B discussed in Section 7.2.

Sol. The given points are $A(0,0)$ and $B(36,15)$.

$A(0,0)$ $B(36,15)$

$\therefore$ Distance $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(36 - 0)^2 + (15 - 0)^2}$
$= \sqrt{1296 + 225}$
$= \sqrt{1521}$
$= 39$

Yes, we can find the distance between the two towns A and B discussed in section 7.2. Ans.

3. Determine if the points $(1, 5), (2, 3)$ and $(-2, -11)$ are collinear.

Sol. The given points are $A(1, 5)$, $B(2, 3)$ and $C(-2, -11)$.

$\therefore$ Distance $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(2 - 1)^2 + (3 - 5)^2}$
$= \sqrt{1 + 4}$
$= \sqrt{5}$

$BC = \sqrt{(-2 - 2)^2 + (-11 - 3)^2}$
$= \sqrt{16 + 196}$
$= \sqrt{212}$

and $CA = \sqrt{[1 - (-2)]^2 + [5 - (-11)]^2}$
$= \sqrt{9 + 256}$
$= \sqrt{265}$

Here, $AB + BC \neq AC$

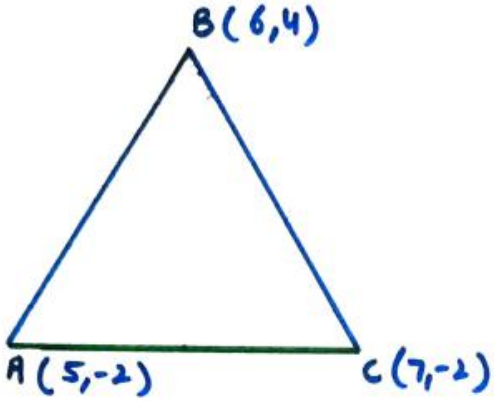
$\therefore$ The points $A(1, 5), B(2, 3)$ and $C(-2, -11)$ are not collinear.

4. Check whether $(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

Sol. Here the points $A(5, -2)$, $B(6, 4)$ and $C(7, -2)$ are the vertices of triangle ABC.

$\therefore$ Distance $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6 - 5)^2 + [4 - (-2)]^2}$
$= \sqrt{1 + 36}$
$= \sqrt{37}$

$BC = \sqrt{(7 - 6)^2 + (-2 - 4)^2}$
$= \sqrt{1 + 36}$
$= \sqrt{37}$



Here, $AB = BC$

$\therefore$ The points $(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle.

5: In a classroom, 4 friends are seated at the points A, B, C and D as shown in Fig. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.

Sol. Here the points are $A(3,4)$, $B(6,7)$, $C(9,4)$ and $D(6,1)$.

$\therefore$ Distance $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-3)^2 + (7-4)^2}$

$= \sqrt{9+9}$

$= 3\sqrt{2}$

$BC = \sqrt{(9-6)^2 + (4-7)^2}$

$= \sqrt{9+9}$

$= 3\sqrt{2}$

$CD = \sqrt{(6-9)^2 + (1-4)^2}$

$= \sqrt{9+9}$

$= 3\sqrt{2}$

$DA = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{9+9}$

$= 3\sqrt{2}$

$AC = \sqrt{(9-3)^2 + (4-4)^2}$

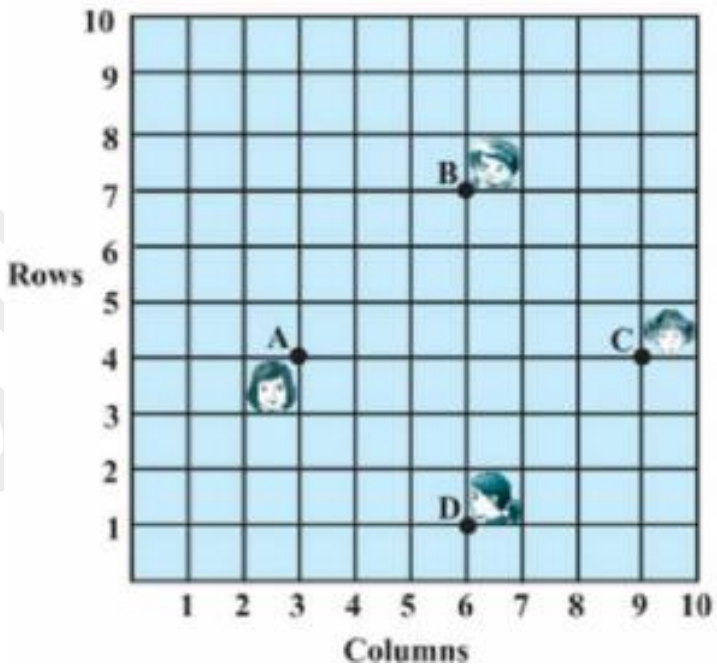
$= \sqrt{36+0}$

$= 6$

$BD = \sqrt{(6-6)^2 + (1-7)^2}$

$= \sqrt{0+36}$

$= 6$



Here, $AB = BC = CD = DA$ and $AC = BD$

Hence, $ABCD$ is a square. So, Champa is correct.

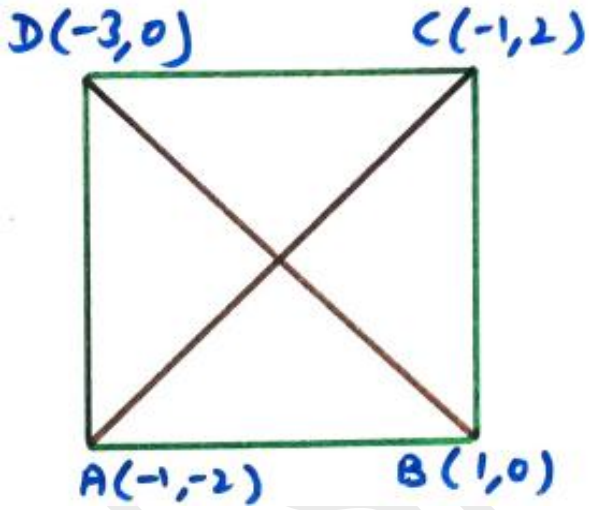
6: Name the type of quadrilateral formed, if any, by the following points, and give reasons for your sol.

(i) $(-1,-2), (1,0), (-1,2), (-3,0)$

(ii) $(-3,5), (3,1), (0,3), (-1,-4)$

(iii) $(4,5), (7,6), (4,3), (1,2)$

Sol.(i) Here the given points are $A(-1,-2), B(1,0), C(-1,2)$ and $D(-3,0)$.



$\therefore$ Distance AB

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[1-(-1)]^2 + [0-(-2)]^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2 + (2-0)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$CD = \sqrt{[-3-(-1)]^2 + (0-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$AC = \sqrt{[-1-(-1)]^2 + [2-(-2)]^2}$

$= \sqrt{0+16}$

$= \sqrt{16}$

$= 4$

$DA = \sqrt{[-1-(-3)]^2 + (-2-0)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$BD = \sqrt{(-3-1)^2 + (0-0)^2}$

$= \sqrt{16+0}$

$= \sqrt{16}$

$= 4$

Here, $AB = BC = CD = DA$ and $AC = BD$

$\therefore$ ABCD is a square.

(ii) Here the given points are $A(-3,5)$, $B(3,1)$, $C(0,3)$ and $D(-1,-4)$.

$AB = \sqrt{[3-(-3)]^2 + (1-5)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{(1-0)^2 + (-4-3)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

$DA = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2} = \sqrt{4+81} = \sqrt{85}$

$AC = \sqrt{[0-(-3)]^2 + (3-5)^2} = \sqrt{9+4} = \sqrt{13}$

$BD = \sqrt{(-1-3)^2 + (-4-1)^2} = \sqrt{16+25} = \sqrt{41}$

Here, $AC + BC = AB$,

It means the point C lies on side AB or A, B, C are collinear.

$\therefore$ The quadrilateral ABCD is not possible.

(iii) Here the given points are $A(4,5)$, $B(7,6)$, $C(4,3)$ and $D(1,2)$.

$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$

$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18}$

$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$

$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18}$

$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+4} = 2$

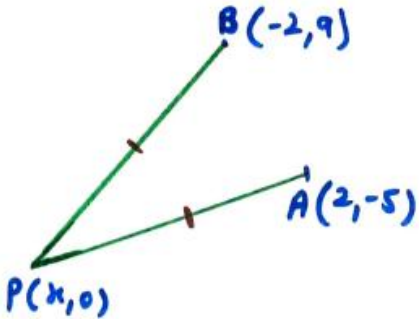
$BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Here, $AB = CD, BC = AD$ and $AC \neq BD$.

$\therefore$ $ABCD$ is a parallelogram.

7: Find the point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$.

Sol. Let $P(x,0)$ be any point on $x-$ axis, which is equidistant from $A(2,-5)$ and $B(-2,9)$.



$\Rightarrow PA = PB$

$\Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$

$\Rightarrow x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$

$\Rightarrow -8x = 81 - 25$

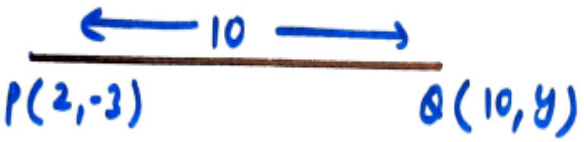
$\Rightarrow -8x = 56$

$\Rightarrow x = -7$

$\therefore P(-7,0)$ is the point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$. Ans.

8: Find the values of y for which the distance between the points $P(2,-3)$ and $Q(10,y)$ is 10 units.

Sol. The distance between $P(2,-3)$ and $Q(10,y)$ is 10 units.



$\Rightarrow PQ = 10$

$\Rightarrow (PQ)^2 = 100$

$\Rightarrow (10-2)^2 + [y-(-3)]^2 = 100$

$\Rightarrow 64 + y^2 + 9 + 6y = 100$

$\Rightarrow y^2 + 6y - 27 = 0$

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y+9) - 3(y+9) = 0$

$\Rightarrow (y+9)(y-3) = 0$

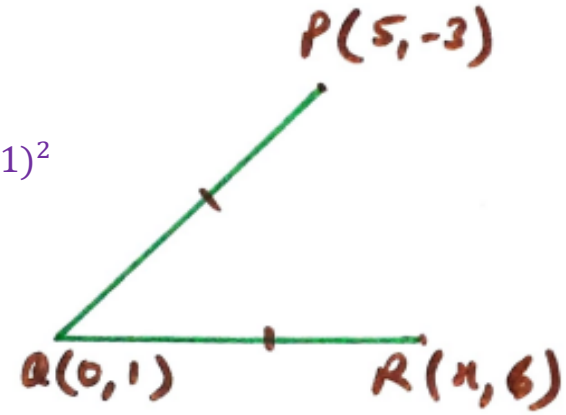
$\Rightarrow (y+9) = 0$ or $(y-3) = 0$

$\Rightarrow y = -9$ or $y = 3$

$\therefore$ The values of y are $y = -9, y = 3$ Ans.

9: If $Q(0,1)$ is equidistant from $P(5,-3)$ and $R(x,6)$, find the values of x. Also find the distances QR and PR.

Sol. $Q(0,1)$ is equidistant from the points $P(5,-3)$ and $R(x,6)$.



Therefore, $PQ = QR$

$\Rightarrow (PQ)^2 = (QR)^2$

$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

For $x = 4$,

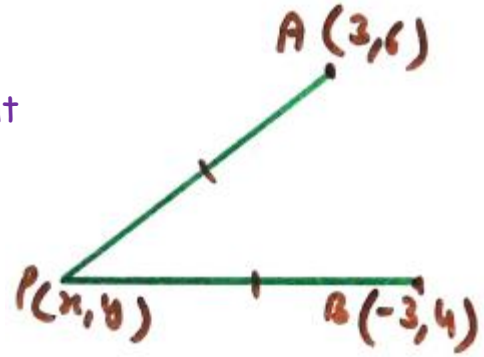
$QR = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

And for $x = -4$,

$QR = \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$

And $PR = \sqrt{(-4-5)^2 + [6-(-3)]^2} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$ Ans.

10. Find a relation between x and y such that the point (x,y) is equidistant from the point $(3,6)$ and $(-3,4)$.



Sol. Let the point $P(x,y)$ is equidistant from $A(3,6)$ and $B(-3,4)$.

$\Rightarrow PA = PB$

$\Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

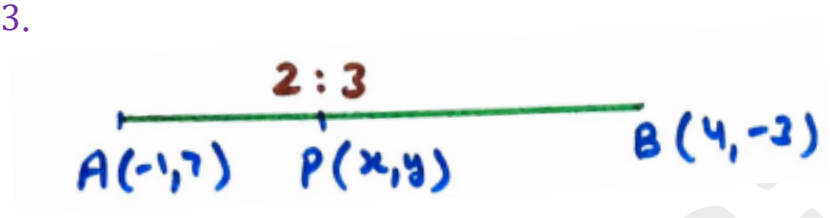
$\Rightarrow -12x - 4y = -20$

$\Rightarrow 3x + y = 5$ Ans.

Exercise 7.2

1. Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio 2:3.

Sol. Let the point P divides the line joining the points $A(-1, 7)$ and $B(4, -3)$ in the ratio $2:3$.



$$\Rightarrow x = \frac{m_1x_2+m_2x_1}{m_1+m_2} \quad \text{and} \quad y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$= \frac{2\times4+3\times(-1)}{2+3} \qquad = \frac{2\times(-3)+3\times7}{2+3}$$

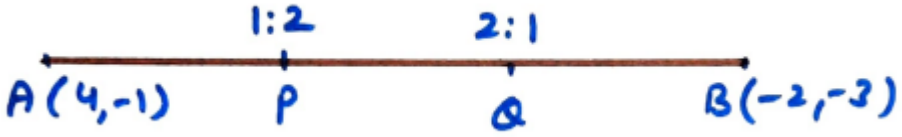
$$= \frac{8-3)}{2+3} \qquad = \frac{-6+21}{2+3}$$

$$= \frac{5}{5} \qquad = \frac{15}{5}$$

$\therefore$ The required point is $(1, 3)$ Ans.

2. Find the coordinates of the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$

Sol. Let the points P and Q be the points of trisection of the line joining $A(4, -1)$ and $B(-2, -3)$.



Here, P divide AB in the ratio $1:2$

$$\Rightarrow x = \frac{m_1x_2+m_2x_1}{m_1+m_2} \quad \text{and} \quad y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$= \frac{1\times(-2)+2\times4}{1+2} \qquad = \frac{1\times(-3)+2\times(-1)}{1+2}$$

$$= \frac{-2+8)}{3} \qquad = \frac{-3-2}{3}$$

$$= \frac{6}{3} \qquad = \frac{-5}{3}$$

Required point P is $(2, -\frac{5}{3})$ Ans.

And Q divide AB in the ratio $2:1$

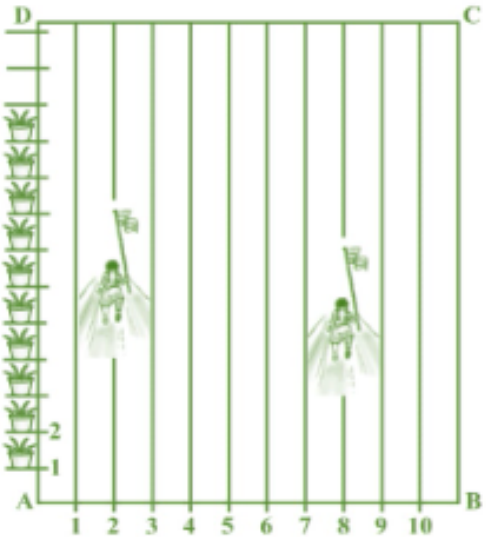
$$\Rightarrow x = \frac{2\times(-2)+1\times4}{2+1} \quad \text{and} \quad y = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$= \frac{-4+4)}{3} \qquad = \frac{-6-1}{3}$$

$$= \frac{0}{3} \qquad = \frac{-7}{3}$$

$\therefore$ Required point Q is $(0, -\frac{7}{3})$ Ans.

3. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs $\frac{1}{4}^{th}$ the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}^{th}$ the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Sol. The coordinates of the flag posted by Niharika $= P\left(2, \frac{1}{4} \times 100\right)$

$= P(2, 25)$

and the coordinates of flag posted by Preet $= Q\left(8, \frac{1}{5} \times 100\right)$

$= Q\ (8, 20)$

Distance between the two flags

$= PQ$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8-2)^2 + (20-25)^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$

The coordinates of the flag posted by Rashmi

$=$ coordinates of mid points of PQ

$= R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= R\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$

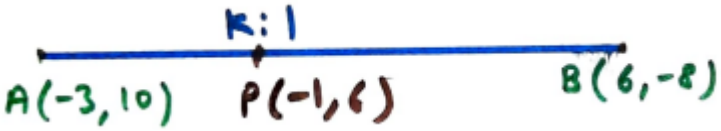
$= R\left(5, \frac{45}{2}\right)$

$= R(5, 22.5)$

Hence, Rashmi should post her flag in fifth line at a distance of 22.5 m.

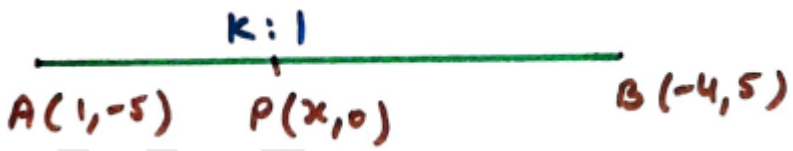
4. Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by $(-1,\ 6)$.

Sol. Let the point $P(-1, 6)$, divides the line segment joining the points $A(-3, 10)$ and $B(6, -8)$ in the ratio $k:1$.



$\Rightarrow x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$
$\Rightarrow -1 = \frac{k\times6+1\times(-3)}{k+1}$
$\Rightarrow -1 = \frac{6k-3}{k+1}$
$\Rightarrow -k-1 = 6k-3$
$\Rightarrow -k-6k = 1-3$
$\Rightarrow -7k = -2$
$\Rightarrow k = \frac{2}{7}$
$\therefore$ The point $P(-1,6)$, divides the line segment joining the points $A(-3,10)$ and $B(6,-8)$ in the ratio $2:7$ Ans.

5. Find the ratio in which the line segment joining $A(1,-5)$ and $B(-4,5)$ is divided by the x-axis. Also find the coordinates of the point of division.
Sol. Let the line segment joining the points $A(1,-5)$ and $B(-4,5)$ is divided by x-axis at the point $P(x,0)$ in the ratio $k:1$.



$\Rightarrow y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$
$\Rightarrow 0 = \frac{k\times5+1\times(-5)}{k+1}$
$\Rightarrow 0 = \frac{5k-5}{k+1}$
$\Rightarrow 0 = 5k-5$
$\Rightarrow 5k = 5$
$\Rightarrow k = 1$
And $x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$
$= \frac{k\times(-4)+1\times1)}{k+1}$
$= \frac{-4\times1+1}{1+1}$
$= \frac{-3}{2}$
$\therefore$ The line segment joining the points $A(1,-5)$ and $B(-4,5)$ is divided by x-axis at point $P\left(-\frac{3}{2},0\right)$ in the ratio $1:1$ Ans.

6. If $(1,2),(4,y),(x,6)$ and $(3,5)$ are the vertices of a parallelogram taken in order, find x and y.
Sol. Here the points $A(1,2),B(4,y),C(x,6)$ and $D(3,5)$ are the vertices of a parallelogram.
We know that the diagonals of a parallelogram bisect each other.

$\Rightarrow$ Coordinates of mid points of AC = Coordinates of mid points of BD

$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

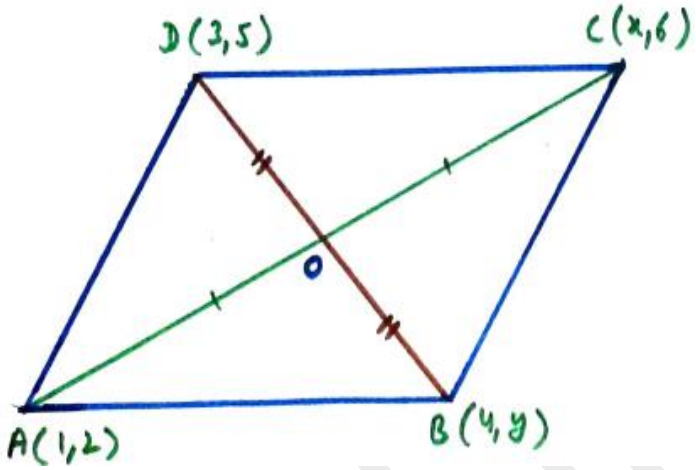
$\Rightarrow \left(\frac{1+x}{2}, \frac{8}{2}\right) = \left(\frac{7}{2}, \frac{5+y}{2}\right)$

On comparing

$\frac{1+x}{2} = \frac{7}{2}$ and $\frac{8}{2} = \frac{5+y}{2}$

$\Rightarrow 1 + x = 7$ and $8 = 5 + y$

$\Rightarrow x = 6$ and $y = 3$



$\therefore$ The values of x and y are 6 and 3 respectively.

7. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is $(2, -3)$ and B is $(1, 4)$.

Sol. Here, AB is the diameter of circle with centre $O(2, -3)$ and the coordinates of $B(1, 4)$.

Let the coordinates of the point A is (x, y).

$\Rightarrow$ Co. of mid-point of AB = Co. of point O

$\Rightarrow \left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (2, -3)$

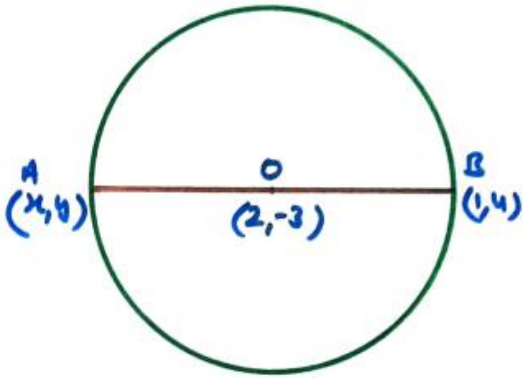
On comparing,

$\frac{x+1}{2} = 2$ and $\frac{y+4}{2} = -3$

$\Rightarrow x + 1 = 4$ and $y + 4 = -6$

$\Rightarrow x = 3$ and $y = -10$

Hence, the coordinates of A is $(3, -10)$ Ans.



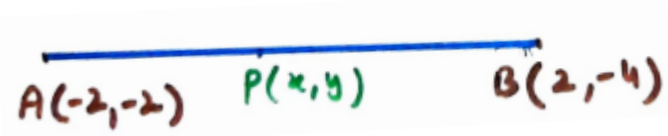
8. If A and B are $(-2, -2)$ and $(2, -4)$, respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Sol. Here $AP = \frac{3}{7}AB$

$\Rightarrow BP = \frac{4}{7}AB$

$\Rightarrow AP : PB = 3 : 4$

$\Rightarrow$ The point P divides the line segment joining the points $A(-2, -2)$ and $B(2, -4)$ in the ratio $3 : 4$.



$\Rightarrow x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$ and $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

$= \frac{3\times2+4\times(-2)}{3+4}$ | $= \frac{3\times(-4)+4\times(-2)}{3+4}$

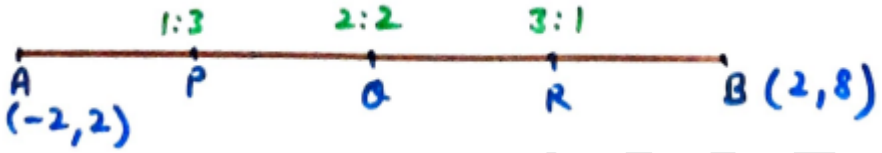
$= \frac{6-8)}{7}$ | $= \frac{-12-8}{7}$

$= \frac{-2}{7}$ | $= \frac{-20}{7}$

$\therefore$ The required point P is $\left(-\frac{2}{7}, -\frac{20}{7}\right)$ Ans.

9. Find the coordinates of the points which divide the line segment joining $A(-2,2)$ and $B(2,8)$ into four equal parts.

Sol. Let the points P, Q and R divides the line segment joining the points $A(-2,2)$ and $B(2,8)$ in to four equal parts.



Here, P divide AB in the ratio $1:3$

$\Rightarrow x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$ and $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

$= \frac{1\times2+3\times(-2)}{1+3}$ | $= \frac{1\times8+3\times2}{1+3}$

$= \frac{2-6}{4}$ | $= \frac{8+6}{4}$

$= \frac{-4}{4}$ | $= \frac{14}{4}$

$\therefore$ The coordinates of point P is $\left(-1, \frac{7}{2}\right)$

Q divide AB in the ratio $2:2$

$\therefore$ The coordinates of point $Q = \left(\frac{2\times2+2\times(-2)}{2+2}, \frac{2\times8+2\times2}{2+2}\right)$

$= \left(\frac{4-4}{4}, \frac{16+4}{4}\right)$

$= \left(\frac{0}{4}, \frac{20}{4}\right)$

$= (0,5)$

And R divide AB in the ratio $3:1$

$\therefore$ The coordinates of point $Q = \left(\frac{3\times2+1\times(-2)}{3+1}, \frac{3\times8+1\times2}{3+1}\right)$

$= \left(\frac{6-2}{4}, \frac{24+2}{4}\right)$

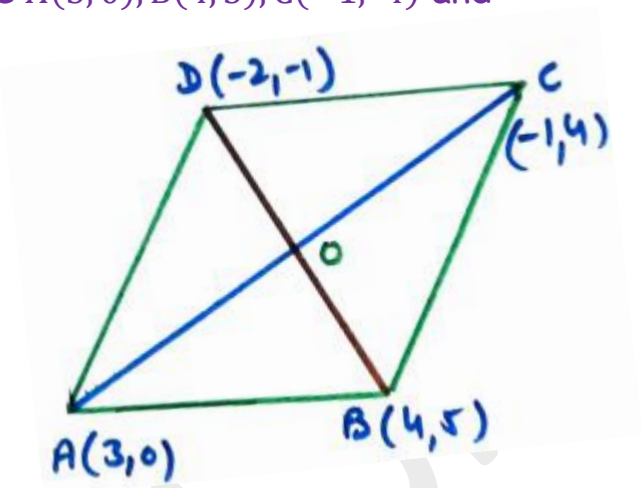
$= \left(\frac{4}{4}, \frac{26}{4}\right)$

$= \left(1, \frac{13}{2}\right)$

$\therefore$ The points $P\left(-1, \frac{7}{2}\right), Q(0,5)$ and $R\left(1, \frac{13}{2}\right)$ divides AB in four equal parts Ans.

10. Find the area of a rhombus if its vertices are $(3,0),(4,5),(-1,4)$ and $(-2,-1)$ taken in order.

Sol. The vertices of rhombus $ABCD$ are $A(3,0), \mathrm{B}(4,5), \mathrm{C}(-1,\ 4)$ and $D(-2,\ -1)$.



Diagonal $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1-3)^2 + (4-0)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Diagonal $BD = \sqrt{(-2-4)^2 + (-1-5)^2}$

$= \sqrt{36+36}$

$= \sqrt{72}$

$= 6\sqrt{2}$

Area of rhombus $= \frac{1}{2}$ (Product of two diagonals)

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$

Area of rhombus is 24 square units. Ans.